Side 9-sætningen: The probability that your vote will make a difference

Pawel Bartoszek

Have you ever wondered when voting on an election day, what the chances are that your vote actually makes a difference? Well, there's an answer, and it involves π!

Let's assume for the sake of simplicity that the number of voters participating in the election is an odd number $n = 2k + 1$. The voters can choose between two options, for example "YES" and "NO".

Let us assume further that they vote independently of one another and that they all vote for each of the options with an equal propability.

Your vote will then count exactly when the vote of the other $2k$ participants are split. The propability of that is

$$P_{\text{Umatter}} = \binom{2k}{k} \frac{1}{2^{2k}} = \frac{(2k)!}{k!k!2^{2k}}.$$

Now, for large k-s we can use Stirling's formula $n! \approx n^n e^{-n} \sqrt{2\pi n}$ to approximate the factorials:

$$P_{\text{Umatter}} \approx \frac{\sqrt{2\pi 2k}(2k)^{2k}e^{-2k}}{\sqrt{2\pi k}k^k e^{-k}\sqrt{2\pi k}k^k e^{-k}2^{2k}} = \frac{2\sqrt{\pi k}(2k)^{2k}e^{-2k}}{2\pi k(2k)^{2k}e^{-2k}} = \frac{1}{\sqrt{\pi k}}.$$

That means that in elections with n participants the chances that your vote will be the deciding one are

$$P_{\text{Umatter}} \approx \sqrt{\frac{2}{(n-1)\pi}}$$

π sure shows up in the strangest places!